The effects of lepton KK modes on the lepton electric dipole moments in the Randall Sundrum scenario

E. O. Iltan [*]

Physics Department, Middle East Technical University
Ankara, Turkey

Abstract

We study the charged lepton electric dipole moments in the Randall Sundrum model where the leptons and the gauge fields are accessible to the extra dimension. We observe that the electron (muon; tau) electric dipole moment reaches to the value of the order of $10^{-26}\, e - cm$ ($10^{-20}\, e - cm$; $10^{-20}\, e - cm$) with the inclusion of the lepton KK modes.

---

[*]E-mail address: eiltan@newton.physics.metu.edu.tr

1 Introduction

The CP violation is among the most interesting physical phenomena and the electric dipole moments (EDMs) of fermions are important tools to understand it since EDMs are driven by the CP violating interaction. There are various experimental and theoretical works done in the literature and the experimental results of the electron, muon and tau EDMs are $d_e = (1.8 \pm 1.2 \pm 1.0) \times 10^{-27} e\, cm$ [1], $d_\mu = (3.7 \pm 3.4) \times 10^{-19} e\, cm$ [2] and $d_\tau = (3.1) \times 10^{-16} e\, cm$ [3], respectively. Furthermore, the experimental upper bound of neutron EDM has been found as $d_N < 1.1 \times 10^{-25} e\, cm$ [4]. From the theoretical point of view, the source of CP violation in the standard model (SM) is the complex Cabbibo-Kobayashi-Maskawa (CKM); lepton mixing matrix in the quark; lepton sector and the calculation of fermion EDMs shows that their numerical values are negligible in the SM. In work [5], the quark EDMs have been estimated as $\sim 10^{-30} (e - cm)$, which is a small quantity since the non-zero contribution exists at least in the three loop level. In order to enhance the fermion EDMs, one needs an alternative source of CP violation and the additional contributions coming from the physics beyond the SM. The multi Higgs doublet models (MHDMs), the supersymmetric model (SUSY) [6] are among the possible models carrying an additional CP phase. The electron EDM has been predicted of the order of the magnitude of $10^{-32} e - cm$ in the two Higgs doublet model (2HDM), including the tree level flavor changing neutral currents (FCNC) [7] and, in this case, the additional CP sources are new complex Yukawa couplings. The EDMs of fermions have been analyzed in [8] and [9], in the 2HDM with the inclusion of non-universal extra dimensions and in the framework of the split fermion scenario. The EDMs of quarks were calculated in the MHDMs, including the 2HDM in [10, 11, 12], the fermion EDMs in the SM, with the inclusion of non-commutative geometry, have been estimated in [13], the lepton EDMs have been studied in the seesaw model in [14], the EDMs of nuclei, deutron, neutron and some atoms have been predicted extensively in [15], limits on the dipole moments of leptons have been analyzed in a left-right symmetric model and in E_6 superstring models in [16].

In the present work, we analyze the charged lepton EDMs in the framework of the 2HDM, with the inclusion of a single extra dimension, respecting the Randall Sundrum (RS1) scenario [17, 18]. The extra dimensions are introduced to solve the hierarchy problem between weak and Planck scales. In the RS1 model, the gravity is localized on a 4D boundary, so called hidden (Planck) brane, and the other fields, including the SM fields, live on another 4D boundary, so called the visible (TeV) brane. The warp factor that is an exponential function of the compactified radius in the extra dimension drives the difference of induced metrics on these

boundaries. With this factor, two effective scales, the Planck scale M_{Pl} and the weak scale m_W, are connected and the hierarchy problem is solved. If the SM fields are accessible to the extra dimension, one obtains a richer phenomenology [19]-[37]. The fermion mass hierarchy can be obtained by considering that the fermions have different locations in the extra dimension and it is induced by the Dirac mass term in the Lagrangian [23, 26, 27, 28]. [29] is devoted to this hierarchy by considering that the Higgs field has an exponential profile around the TeV brane and [30] is devoted to an extensive work on the bulk fields in various multi-brane models. The different locations of the fermion fields in the extra dimension can ensure the flavor violation (FV) and it is carried by the Yukawa interactions, coming from the SM Higgs-fermion-fermion vertices. The fermion localization in the RS1 background has been applied to the high precision measurements of top pair production at the ILC [35] and the various experimental FCNC constraints and the electro weak precision tests for the location parameters of the fermions in the extra dimension are analyzed in [36, 37].

Here, we study the charged lepton EDMs in the case that the leptons and gauge fields are accessible to the extra dimension with localized leptons in the RS1 background. We observe that the d_e (d_μ; d_τ) reach to the values of the order of $10^{-26}\,e-cm$ ($10^{-20}\,e-cm$; $10^{-20}\,e-cm$) with the inclusion of the KK modes.

The paper is organized as follows: In Section 2, we present EDMs of charged leptons in the RS1 scenario, in the 2HDM. Section 3 is devoted to discussion and our conclusions. In Appendix section, we present how to construct the SM fermions and their KK modes.

2 Electric dipole moments of charged leptons in the two Higgs doublet model, in the RS1 scenario.

The fermion EDM arises from the CP violating fermion-fermion-photon effective interaction and, therefore, their experimental and theoretical search ensure considerable information about the nature of the CP violation. In the SM, the CP violation is driven by the complex CKM matrix elements in the quark sector and a possible lepton mixing matrix in the lepton sector. The tiny theoretical values of fermion EDMs in the SM force one to go beyond and the extension of the Higgs sector with FCNCs at tree level is one of the possibility in order to enhance their theoretical values. In the present work, we consider the 2HDM, which allows the FCNC at tree level with the complex Yukawa couplings[1]. The additional effect coming from the extra

---

[1] Here, we assume that the CKM type matrix in the lepton sector does not exist, the charged flavor changing (FC) interactions vanish and the lepton FV comes from the internal new neutral Higgs bosons, h^0 and A^0.

dimension(s) is another possibility to enhance the CP violation and, here, we consider the RS1 scenario with localized charged leptons in the extra dimension. The RS1 background is curved and the corresponding metric reads

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \,, \tag{1}$$

where $\sigma = k\,|y|$ with the bulk curvature constant k and the exponential $e^{-k\,|y|}$, with $y = R\,|\theta|$, is the warp factor which drives the hierarchy and rescales all mass terms on the visible brane for $\theta = \pi$. Here R is the compactification radius in the extra dimension that is compactified onto S^1/Z_2 orbifold with two boundaries, the hidden (Planck) brane and the visible (TeV) brane. In the RS1 model, all SM fields live in the visible brane, however, the gravity is accessible to the bulk and it is considered to be localized on the hidden brane. With the assumption that the gauge fields and the fermions are also accessible to the extra dimension, the particle spectrum is extended and the physics becomes richer. In the present work, we consider this scenario with the addition of Z_2 invariant[2] Dirac mass term to the lagrangian of bulk fermions which results in the fermion localization in the extra dimension [20, 22, 23, 25, 26, 28, 29]

$$\mathcal{S}_m = -\int d^4x \int dy \sqrt{-g}\, m(y)\, \bar{\psi}\psi \,, \tag{2}$$

where $m(y) = m\frac{\sigma'(y)}{k}$ with $\sigma'(y) = \frac{d\sigma}{dy}$ and $g = Det[g_{MN}] = e^{-8\sigma}$, $M, N = 0, 1, ..., 4$. Here the bulk fermion is expanded as

$$\psi(x^\mu, y) = \frac{1}{\sqrt{2\,\pi\,R}} \sum_{n=0}^{\infty} \psi^{(n)}(x^\mu)\, e^{2\sigma}\, \chi_n(y) \,, \tag{3}$$

with the normalization

$$\frac{1}{2\,\pi\,R} \int_{-\pi\,R}^{\pi\,R} dy\, e^\sigma\, \chi_n(y)\, \chi_m(y) = \delta_{nm} \,. \tag{4}$$

Using the Dirac equation and the normalization condition, the zero mode fermion is obtained as follows:

$$\chi_0(y) = N_0\, e^{-r\,\sigma} \,, \tag{5}$$

where $r = m/k$ and the normalization constant N_0 reads

$$N_0 = \sqrt{\frac{k\,\pi\,R\,(1 - 2\,r)}{e^{k\,\pi\,R\,(1 - 2\,r)} - 1}} \,. \tag{6}$$

---

[2] The fermions have two possible transformation properties under the orbifold Z_2 symmetry, $Z_2\psi = \pm\gamma_5\psi$ and the combination $\bar{\psi}\psi$ is odd. The Z_2 invariant mass term is obtained if the Z_2 odd scalar field is coupled to the combination $\bar{\psi}\psi$.

On the other hand

$$\chi_0'(y) = e^{-\frac{\sigma}{2}} \chi_0(y) \,, \tag{7}$$

is the appropriately normalized solution and it is localized in the extra dimension. The parameter r is responsible for the localization and for $r > \frac{1}{2}$ ($r < \frac{1}{2}$) it is localized near the hidden (visible) brane.

The action which is responsible for the charged lepton EDMs in the RS1 background is:

$$\mathcal{S}_Y = \int d^5x \sqrt{-g} \left(\xi^E_{5ij} \, \bar{l}_{iL} \phi_2 E_{jR} + h.c. \right) \delta(y - \pi R) \,, \tag{8}$$

where L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_2 is the new scalar doublet, l_{iL} (E_{jR}) are lepton doublets (singlets), ξ^E_{5ij}, with family indices i, j , are the complex Yukawa couplings in five dimensions and they induce the FV interactions in the lepton sector. Here, we assume that the Higgs doublet ϕ_1, living on the visible brane, has non-zero vacuum expectation value to ensure the ordinary masses of the gauge fields and the fermions, however, the second doublet, that lies also on the visible brane, has no vacuum expectation value:

$$\phi_1 = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix} \right] \,;\, \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ H_1 + iH_2 \end{pmatrix} \,, \tag{9}$$

and

$$< \phi_1 > = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \,;\, < \phi_2 > = 0 \,. \tag{10}$$

The gauge and CP invariant Higgs potential which spontaneously breaks $SU(2) \times U(1)$ down to $U(1)$ reads:

$$\begin{aligned} V(\phi_1, \phi_2) &= c_1(\phi_1^+\phi_1 - v^2/2)^2 + c_2(\phi_2^+\phi_2)^2 \\ &+ c_3[(\phi_1^+\phi_1 - v^2/2) + \phi_2^+\phi_2]^2 + c_4[(\phi_1^+\phi_1)(\phi_2^+\phi_2) - (\phi_1^+\phi_2)(\phi_2^+\phi_1)] \\ &+ c_5[Re(\phi_1^+\phi_2)]^2 + c_6[Im(\phi_1^+\phi_2)]^2 + c_7 \,, \end{aligned} \tag{11}$$

with constants c_i, $i = 1, ..., 7$. The choice eq.(10) and the potential eq.(11) leads to the fact that the SM particles are collected in the first doublet and the new particles in the second one. This is the case that no mixing occurs between CP-even neutral Higgs bosons H^0 and h^0 in the tree level and H_1 and H_2 in eq. (9) are obtained as the mass eigenstates h^0 and A^0, respectively.

The lepton doublet and singlet fields in eq. (8) is expanded to their KK modes as

$$l_{iL}(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} e^{2\sigma} l_{iL}^{(0)}(x^\mu) \chi_{i\,L0}(y)$$

4

$$+ \frac{1}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} e^{2\sigma} \left(l_{iL}^{(n)}(x^\mu) \chi_{iLn}^l(y) + l_{iR}^{(n)}(x^\mu) \chi_{iRn}^l(y) \right),$$

$$E_{jR}(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} e^{2\sigma} E_{jR}^{(0)}(x^\mu) \chi_{jR0}(y)$$

$$+ \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} e^{2\sigma} \left(E_{jR}^{(n)}(x^\mu) \chi_{jRn}^E(y) + E_{jL}^{(n)}(x^\mu) \chi_{jLn}^E(y) \right), \tag{12}$$

where the zero mode leptons $\chi_{iL0}(y)$ and $\chi_{jR0}(y)$ are given in eq. (5) with the replacements $r \to r_{iL}$ and $r \to r_{iR}$, respectively. For the effective Yukawa coupling ξ_{ij}^E, we integrate out the Yukawa interaction eq.(8) over the fifth dimension by taking the zero mode lepton doublets, singlets, and neutral Higgs fields $S = h^0, A^0$:

$$V_{RL\,ij}^0 = \frac{\xi_{5\,ij}^E}{2\pi R} \int_{-\pi R}^{\pi R} dy\, \chi_{iR0}(y)\, \chi_{jL0}(y)\, \delta(y - \pi R)$$

$$= \frac{e^{-k\pi R(r_{iR}+r_{jL})} k \sqrt{(1 - 2\,r_{iR})(1 - 2\,r_{jL})}}{\sqrt{(e^{k\pi R(1-2\,r_{iR})} - 1)(e^{k\pi R(1-2\,r_{jL})} - 1)}} \xi_{5\,ij}^E. \tag{13}$$

Here, we embed the vertex factor $V_{RL\,(LR)\,ij}^0$ into the coupling $\xi_{ij}^E \left((\xi_{ij}^E)^\dagger \right)$ and fix the numerical value of $\xi_{ij}^E \left((\xi_{ij}^E)^\dagger \right)$ by assuming that the coupling $\xi_{5\,ij}^E$ in five dimension is flavor dependent. In this case the hierarchy of new Yukawa couplings is not related to the lepton field locations. On the other hand, we need the vertex factors due to S-KK mode charged lepton-charged lepton couplings $V_{LR\,(RL)\,ij}^n$ since charged lepton KK modes exist in the internal line (see fig.1). After the integration of the Yukawa interaction eq.(8) over the fifth dimension, the S-zero mode lepton singlet (doublet)-KK mode lepton doublet (singlet) vertex factor (see appendix for the construction of KK mode charged lepton doublets and singlets) reads

$$V_{RL\,(LR)\,ij}^n = \frac{\xi_{5\,ij}^E \left((\xi_{5\,ij}^E)^\dagger \right)}{2\pi R} \int_{-\pi R}^{\pi R} dy\, \chi_{iR0\,(iL0)}(y)\, \chi_{jLn\,(jRn)}(y)\, \delta(y - \pi R), \tag{14}$$

and

$$V_{RL\,ij}^n = \frac{N_{Ln}\, e^{k\pi R(1/2 - r_{iR})} \left(J_{\frac{1}{2}-r_{jL}}(e^{k\pi R} x_{nL}) + c_L\, Y_{\frac{1}{2}-r_{jL}}(e^{k\pi R} x_{nL}) \right)}{\pi R \sqrt{\frac{e^{k\pi R(1-2\,r_{iR})} - 1}{k\pi R(1-2\,r_{iR})}}} \xi_{5\,ij}^E,$$

$$V_{LR\,ij}^n = \frac{N_{Rn}\, e^{k\pi R(1/2 - r_{iL})} \left(J_{\frac{1}{2}+r_{jR}}(e^{k\pi R} x_{nR}) + c_R\, Y_{\frac{1}{2}+r_{jR}}(e^{k\pi R} x_{nR}) \right)}{\pi R \sqrt{\frac{e^{k\pi R(1-2\,r_{iL})} - 1}{k\pi R(1-2\,r_{iL})}}} (\xi_{5\,ij}^E)^\dagger, \tag{15}$$

where c_L (c_R) is given in eq.(29) (eq.(32)) with the replacements $r \to r_{jL}$ ($r \to r_{jR}$). Here the effective S-zero mode lepton singlet (doublet)-KK mode lepton doublet (singlet) coupling

$\xi_{ij}^{E\,n} \, (\xi_{ij}^{E\,n})^\dagger$ reads

$$\xi_{ij}^{E\,n} \left((\xi_{ij}^{E\,n})^\dagger \right) = \frac{V_{RL\,(LR)\,ij}^n}{V_{RL\,(LR)\,ij}^0} \, \xi_{ij}^E \; . \tag{16}$$

Notice that the strengths of S-KK mode charged lepton-charged lepton couplings are regulated by the locations of the lepton fields and we use two different sets, Set I and Set II[3].

	SET I		SET II	
	r_L	r_R	r_L	r_R
e	-0.4900	0.8800	-1.0000	0.8860
μ	-0.4900	0.7160	-1.0000	0.7230
τ	-0.4900	0.6249	-1.0000	0.6316

Table 1: Two possible locations of charged lepton fields. Here r_L and r_R are left handed and right handed lepton field location parameters, respectively.

The effective EDM interaction for a fermion f is reads

$$\mathcal{L}_{EDM} = id_f \, \bar{f} \, \gamma_5 \, \sigma^{\mu\nu} \, f \, F_{\mu\nu} \; , \tag{17}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor, 'd_f' is EDM of the fermion and it is a real number by hermiticity (see Fig. 1 for the 1-loop diagrams which contribute to the EDMs of fermions).

Now, we present the charged lepton EDMs with the addition of KK modes in the framework of the RS1 scenario. Since there is no CKM type lepton mixing matrix according to our assumption, only the neutral Higgs part gives a contribution to their EDMs and l-lepton EDM 'd_l' ($l = e, \mu, \tau$) can be calculated as a sum of contributions coming from neutral Higgs bosons h_0 and A_0. For $l = e, \mu$ and we get[4],

$$
\begin{aligned}
d_l &= -\frac{iG_F}{\sqrt{2}} \frac{e}{32\pi^2} Q_\tau \Bigg\{ \frac{1}{m_\tau} \left((\bar{\xi}_{N,l\tau}^{E\,*})^2 - (\bar{\xi}_{N,\tau l}^E)^2 \right) \left((F_1(y_{h^0}) - F_1(y_{A_0})) \right) \\
&\quad + \sum_{n=1}^{\infty} \left((\bar{\xi}_{N,l\tau}^{E\,n\,*})^2 - (\bar{\xi}_{N,\tau l}^{E\,n})^2 \right) \left(G(y_{nL,h^0}, y_{nR,h^0}) - G(y_{nL,A^0}, y_{nR,A^0}) \right) \Bigg\},
\end{aligned}
\tag{18}
$$

where

$$
\begin{aligned}
G(y_{nL,S}, y_{nR,S}) &= G_1(y_{nL,S}, y_{nR,S}) + G_1(y_{nR,S}, y_{nL,S}) + G_2(y_{nL,S}, y_{nR,S}) \\
&\quad + G_2(y_{nR,S}, y_{nL,S}) \, , \tag{19}
\end{aligned}
$$

---

[3]In this scenario, the source of FV is not related to the different locations of the fermion fields in the extra dimension (see [33, 34]) but it is carried by the new Yukawa couplings in four dimensions and the additional effect due to the extra dimension is the enhancement in the physical quantities of the processes studied.

[4]In the following we use the dimensionful coupling $\bar{\xi}_{N,ij}^E$ in four dimensions, with the definition $\xi_{N,ij}^E = \sqrt{\frac{4G_F}{\sqrt{2}}} \, \bar{\xi}_{N,ij}^E$ where N denotes the word "neutral".

with

$$F_1(w) \quad = \quad \frac{w \left(3 - 4\,w + w^2 + 2\,ln\,w\right)}{(-1+w)^3} \;,$$

(20)

and

$$G_1(y_{nL,S}, y_{nR,S}) \quad = \quad \frac{y_{nR,S}\left(m_{nL}\left(y_{nR,S}-1\right)y_{nR,S} - m_{nR}\left(y_{nL,S}\left(y_{nR,S}-2\right)+y_{nR,S}\right)\right)}{m_S^2\left(y_{nR,S}-y_{nL,S}\right)^2\left(1-y_{nR,S}\right)^2}\,ln\,y_{nR,S}\;,$$

$$G_1(y_{nR,S}, y_{nL,S}) \quad = \quad G_1(y_{nL,S}, y_{nR,S})|_{y_{nL,S}\Leftrightarrow y_{nR,S};\,m_{nL}\Leftrightarrow m_{nR}}\;,$$

$$G_2(y_{nL,S}, y_{nR,S}) \quad = \quad \frac{y_{nL,S}\,m_{nL}}{m_S^2\left(y_{nR,S}-y_{nL,S}\right)\left(1-y_{nL,S}\right)}\;,$$

$$G_2(y_{nR,S}, y_{nL,S}) \quad = \quad G_2(y_{nL,S}, y_{nR,S})|_{y_{nL,S}\Leftrightarrow y_{nR,S};\,m_{nL}\Leftrightarrow m_{nR}}\;.$$

(21)

The tau lepton EDM reads

$$\begin{aligned}
d_\tau \quad = \quad & -\frac{iG_F}{\sqrt{2}}\frac{e}{16\pi^2}\,Q_\tau\Bigg\{\frac{1}{m_\tau}\left((\bar{\xi}_{N,\tau\tau}^{E\,*})^2 - (\bar{\xi}_{N,\tau\tau}^{E})^2\right)\int_0^1 dx \int_0^{1-x} dy\left((x-1)\left(\frac{y_{h^0}}{L_{h^0}} - \frac{y_{A^0}}{L_{A^0}}\right)\right. \\
& + \sum_{n=1}^{\infty}\int_0^1 dx \int_0^{1-x} dy\left((\bar{\xi}_{N,\tau\tau}^{E\,n})^2\left(m_{nR}\,y + m_{nL}\left(1-x-y\right)\right)\left(\frac{1}{m_{h^0}^2\,L_{n,h^0}} - \frac{1}{m_{A^0}^2\,L_{n,A^0}}\right)\right. \\
& - \left.\left.(\bar{\xi}_{N,\tau\tau}^{E\,n\,*})^2\left(m_{nL}\,y + m_{nR}\left(1-x-y\right)\right)\left(\frac{1}{m_{h^0}^2\,L'_{n,h^0}} - \frac{1}{m_{A^0}^2\,L'_{n,A^0}}\right)\right)\right\}\;,
\end{aligned}$$

(22)

where

$$\begin{aligned}
L_S \quad &= \quad x + (x-1)^2\,y_S\;, \\
L_{n,S} \quad &= \quad x + x\,(x-1)\,y_S + y\,y_{nR,S} + (1-x-y)\,y_{nL,S}\;, \\
L'_{n,S} \quad &= \quad L_{n,S}|_{y_{nL,S}\Leftrightarrow y_{nR,S}}\;.
\end{aligned}$$

(23)

Here $y_S = \frac{m_\tau^2}{m_S^2}$, $y_{nL\,(nR),S} = \frac{m_{nL\,(nR)}^2}{m_S^2}$ and Q_τ is the tau lepton charge. In eqs. (18) and (22) we take into account only the internal τ-lepton contribution, respecting our assumption that the Yukawa couplings $\bar{\xi}_{N,ij}^{E}$, $i,j = e,\mu$, are small compared to $\bar{\xi}_{N,\tau i}^{E}\,i = e,\mu,\tau$ (see Discussion section for details). Notice that, we make our calculations in arbitrary photon four momentum square q^2 and take $q^2 = 0$ at the end. For the Yukawa couplings we used the parametrization

$$\bar{\xi}_{N,\tau l}^{E\,n} = |\bar{\xi}_{N,\tau l}^{E\,n}|\,e^{i\theta_l}\;,$$

(24)

where $l = e,\mu,\tau$. Here, θ_l is the CP violating parameter which is the source of the lepton EDM[5].

---

[5]The Yukawa factors in eq.(18) can be written as

$$((\bar{\xi}_{N,l\tau}^{E\,n\,*})^2 - (\bar{\xi}_{N,\tau l}^{E\,n})^2) = -2\,i sin\,2\theta_l\,|\bar{\xi}_{N,\tau l}^{E\,n}|^2\;.$$

(25)

3 Discussion

In this section, we study the effects lepton KK modes on the EDMs of charged leptons in the framework of the RS1 scenario, with extended Higgs sector. The source of fermion EDMs is the CP violating interaction that is arisen from a CP violating phase. Here, we assume that this phase comes from the complex Yukawa couplings appearing in the tree level fermion-fermion-new Higgs interactions, in the framework of the 2HDM. In the case of charged leptons, the leptonic complex Yukawa couplings $\bar{\xi}^E_{N,ij}, i, j = e, \mu, \tau$ are responsible for the EDMs and they are free parameters in the model considered. Here, we expect that the Yukawa couplings $\bar{\xi}^E_{N,ij}, i, j = e, \mu$ are weak compared to $\bar{\xi}^E_{N,\tau i} i = e, \mu, \tau$ and the couplings $\bar{\xi}^E_{N,ij}$ in four dimensions are symmetric with respect to the indices i and j. Finally, we consider that the coupling $\bar{\xi}^E_{N,\tau e}$, $(\bar{\xi}^E_{N,\tau \mu}, \bar{\xi}^E_{N,\tau \tau})$ is dominant among the others. This is the case that the tau lepton and its KK mode appear in the internal line (see Fig.1). The numerical value of the coupling $\bar{\xi}^E_{N,\tau \mu}$ is chosen by respecting the experimental uncertainty, 10^{-9}, in the measurement of the muon anomalous magnetic moment [38] (see [39] for details)[6]. This upper limit and the experimental upper bound of BR of $\mu \to e\gamma$ decay, BR $\leq 1.2 \times 10^{-11}$, can give clues about the numerical value of the coupling $\bar{\xi}^E_{N,\tau e}$ (see [7]) and we take it of the order of $10^{-2} \, (GeV)$. For the coupling $\bar{\xi}^E_{N,\tau \tau}$ there is no stringent prediction and we consider an intermediate value which is greater than the coupling $\bar{\xi}^E_{N,\tau \mu}$. For the CP violating parameter which drives the EDM interaction we choose the range, $0.1 \geq \sin \theta_{e \, (\mu, \, \tau)} \geq 0.7$.

In the present work, we study the charged lepton EDMs in the RS1 background with the assumption that the leptons are also accessible to the extra dimension. The inclusion of extra dimensions brings additional contributions which come from the KK modes of leptons in the 4D effective theory after the compactification. Here, we consider that the lepton fields are localized in the extra dimension with the help of the Dirac mass term $m_l = r\sigma'$, $\sigma = k \, |y|$ (eq.(2)) in the action. This is the case that the SM leptons, the right and left handed parts are located in the extra dimension with exponential profiles (see eq.(5)) which makes it possible to explain the different flavor mass hierarchy (see Appendix section for the construction of the SM fields and their masses).

The gauge sector of the model should live in the extra dimension necessarily if the leptons exist in the bulk and, therefore, their KK modes appear after the compactification of the extra dimension. These KK modes result in additional FCNC effects at tree level coming from

---

[6]In [39], the upper limit of the coupling $\bar{\xi}^E_{N,\tau \mu}$ is estimated as $\sim 30 \, (GeV)$ in the framework of the 2HDM and here, we take the numerical value which is less than this quantity.

the couplings with charged leptons and they should be suppressed even for low KK masses, by choosing the lepton location parameters c_L (c_R) appropriately (see the discussion given in [36, 37]). Here, we use two different sets of locations of charged leptons (Table 1) in order to obtain the masses of different flavors and we verify the various experimental FCNC constraints with KK neutral gauge boson masses as low as few TeVs. In both sets, we estimate the right handed locations of leptons by choosing the left handed charged lepton locations as the same. For the case that the left handed charged lepton locations near to the visible brane (see set II), the strengths of the couplings of leptons with the new Higgs doublet living in the 4D brane becomes stronger and, therefore, the physical quantities related to these couplings enhance.

Throughout our calculations we use the input values given in Table (2). Furthermore, the

Parameter	Value
m_μ	0.106 (GeV)
m_τ	1.78 (GeV)
m_{h^0}	100 (GeV)
m_{A^0}	200 (GeV)
G_F	$1.1663710^{-5}(GeV^{-2})$

Table 2: The values of the input parameters used in the numerical calculations.

curvature parameter k and the compactification radius R are the additional free parameters of the theory. Here we take $k R = 10.83$ and consider in the region $10^{17} \leq k \leq 10^{18}$ (see the discussion in appendix and the work [28]).

In Fig.2, we present the parameter k dependence of the electron EDM d_e, for $\bar{\xi}^E_{N,\tau e} = 0.01\,(GeV)$ and two different values of the CP violating parameter $sin\,\theta$. Here, the lower-upper solid (dashed, small dashed) line represents the d_e for $sin\,\theta = 0.1 - 0.5$ without KK modes (with KK modes set I, II). It is observed that the d_e is of the order of $10^{-29} - 10^{-28}\,e - cm$ for $sin\,\theta = 0.1 - 0.5$ without KK modes. The inclusion of the KK modes enhances the d_e 50 times-two orders for the set I-II for the values of the curvature scale $k \sim 10^{17}\,(GeV)$ and this enhancement becomes weak for $k \sim 10^{18}\,(GeV)$. For the set II, the enhancement in the d_e is almost two times larger compared to the set I case. This is due to the fact that the left handed leptons (zero and KK modes) are near to the visible brane and their couplings to the new Higgs scalars become stronger for set II case. The experimental upper limit is $d_e = (1.8 \pm 1.2 \pm 1.0) \times 10^{-27}e\,cm$ and this numerical value can be reached even for small values of the CP parameter $sin\,\theta \sim 0.1$ with the inclusion of charged lepton KK modes.

Fig. 3 is devoted to the parameter k dependence of the d_μ, for $\bar{\xi}^E_{N,\tau\mu} = 1\,(GeV)$ and two

different values of the CP violating parameter $sin\,\theta$. Here, the lower-upper solid (dashed, small dashed) line represents the d_μ for $sin\,\theta = 0.1 - 0.5$ without KK modes (with KK modes set I, II). We observe that the d_μ is of the order of $10^{-25} - 10^{-24}\,e - cm$ for $sin\,\theta = 0.1 - 0.5$ without KK modes. With the inclusion of the KK modes the d_μ increases to the values of the order of 10^{-23}; $2.0 \times 10^{-23}\,e - cm$ for $sin\,\theta = 0.1$ and 5.0×10^{-23}; $10^{-22}\,e - cm$ for $sin\,\theta = 0.5$ in the case of the set I; set II, for the values of the curvature scale $k \sim 10^{17}\,(GeV)$. With the choice of $\bar{\xi}^E_{N,\tau\mu} = 10\,(GeV)$, which is the numerical value near to the upper limit that is obtained by respecting the experimental uncertainty, 10^{-9}, in the measurement of the muon anomalous magnetic moment (see [39]), the d_μ is reached to the value $10^{-20}\,e - cm$ for $sin\,\theta \sim 0.5$, with the inclusion of charged lepton KK modes in the case of set II and $k \sim 10^{17}\,(GeV)$. This is a numerical value near to the experimental upper limit $d_\mu = (3.7 \pm 3.4) \times 10^{-19}\,e - cm$.

Fig.4 represents the parameter k dependence of the d_τ, for $\bar{\xi}^E_{N,\tau\tau} = 10\,(GeV)$ and two different values of the CP violating parameter $sin\,\theta$. Here, the lower-upper solid (dashed, small dashed) line represents the d_τ for $sin\,\theta = 0.1 - 0.5$ without KK modes (with KK modes set I, II). Here it is observed that the d_τ is of the order of $10^{-23} - 10^{-22}\,e - cm$ for $sin\,\theta = 0.1 - 0.5$ without KK modes. The inclusion of the KK modes causes that d_τ is enhanced to the values of the order of 10^{-21}; $2.0 \times 10^{-21}\,e - cm$ for $sin\,\theta = 0.1$ and 5.0×10^{-21}; $10^{-20}\,e - cm$ for $sin\,\theta = 0.5$ in the case of the set I; set II, for the values of the curvature scale $k \sim 10^{17}\,(GeV)$. The experimental upper limit of d_τ, $|d_\tau| < (3.1) \times 10^{-16}\,e - cm$, is almost two orders far from the theoretical value obtained, even with the strong coupling $\bar{\xi}^E_{N,\tau\tau} \sim 100\,(GeV)$ and it needs more sensitive experimental measurements.

Now, we analyze the CP violating parameter $sin\,\theta$ dependence of the charged lepton EDMs, for completeness.

In Fig.5, we present the parameter $sin\,\theta$ dependence of the d_e; d_μ; d_τ, for $\bar{\xi}^E_{N,\tau e} = 0.01\,(GeV)$; $\bar{\xi}^E_{N,\tau\mu} = 1.0\,(GeV)$; $\bar{\xi}^E_{N,\tau\tau} = 10\,(GeV)$ and for $k = 10^{18}\,(GeV)$. Here, the lower-intermediate-upper solid (dashed, small dashed) line represents the d_e-d_μ-d_τ without KK modes (with KK modes set I, II). Here it is observed that the EDMs are not so much sensitive to the location of lepton fields in the bulk for the large values of the curvature parameter k, $k \sim 10^{18}\,(GeV)$ and the enhancements in the EDMs are %6 (%12) for set I (set II). This sensitivity becomes weak for the small value of the CP violating parameter $sin\,\theta$.

With the more accurate experimental investigation of the charged lepton EDMs, it will be possible to understand the mechanism behind the CP violation and one will get powerful information about the effects of warped extra dimensions, if they exist.

4 Appendix

The SM fermions are constructed by considering the $SU(2)_L$ doublet ψ_L and the singlet ψ_R, satisfying two separate Z_2 projection conditions: $Z_2\psi_R = \gamma_5\psi_R$ and $Z_2\psi_L = -\gamma_5\psi_L$ (see for example [20]). The zero mode fermions can get mass through the Z_2 invariant left handed fermion-right handed fermion-Higgs interaction, $\bar{\psi}_R\psi_L H$[7] and, one gets the location parameters of the left and the right handed parts of fermions in order to obtain the current masses of fermions of different flavors. If we consider that the SM Higgs field lives on the visible brane, the masses of fermions are calculated by using the integral

$$m_i = \frac{1}{2\pi R} \int_{-\pi R}^{\pi R} dy\, \lambda_5\, \chi_{iL0}(y)\, \chi_{iR0}(y) <H> \delta(y - \pi R), \tag{26}$$

where λ_5 is the coupling in five dimensions and it can be parametrized in terms of the one in four dimensions, the dimensionless coupling λ, $\lambda_5 = \lambda/\sqrt{k}$. Here the expectation value of the Higgs field $<H>$ reads $<H> = v/\sqrt{k}$ where v is the vacuum expectation value, $v = 0.043\, M_{Pl}$, in order to provide the measured gauge boson masses [28] and choose $k\,R = 10.83$ in order to get the correct effective scale on the visible brane, i.e., $M_W = e^{-\pi k R} M_{pl}$ is of the order of TeV.

Since the EDMs of fermions exist at least in the one loop level, there appears the S-charged lepton-KK charged lepton vertices. The Z_2 the projection condition $Z_2\psi = -\gamma_5\psi$, used to construct the left handed fields on the branes and as a result, the left handed zero and KK modes appear, the right handed KK modes disappear on the branes. Here the boundary conditions coming from the Dirac mass term in the action eq.(2):

$$\left(\frac{d}{dy} - m\right) \chi_{iLn}^l(y_0) = 0$$
$$\chi_{iRn}^l(y_0) = 0, \tag{27}$$

where $y_0 = 0$ or πR. The left handed lepton $\chi_{i\,Ln}^l(y)$ that lives on the visible brane is obtained

$$\chi_{iLn}^l(y) = N_{Ln}\, e^{\sigma/2}\left(J_{\frac{1}{2}-r}(e^\sigma x_{nL}) + c_L\, Y_{\frac{1}{2}-r}(e^\sigma x_{nL})\right), \tag{28}$$

by using the Dirac equation for KK mode leptons. Here the constant c_L is

$$c_L = -\frac{J_{-r-\frac{1}{2}}(x_{nL})}{Y_{-r-\frac{1}{2}}(x_{nL})}. \tag{29}$$

where N_{Ln} is the normalization constant and $x_{nL} = \frac{m_{Ln}}{k}$. The functions $J_\beta(w)$ and $Y_\beta(w)$ appearing in eq.(28) are the Bessel function of the first kind and of the second kind, respectively.

---

[7]Here, we consider different location parameters r for each left handed and right handed part of different flavors.

The right handed zero mode fields on the branes can be constructed by considering the Z_2 projection condition $Z_2\psi = \gamma_5\psi$ and this ensures that the right handed zero mode appears, the right (left) handed KK modes appear (disappear) on the branes with the boundary conditions:

$$\left(\frac{d}{dy} + m\right)\chi_{iRn}^{E}(y_0) = 0$$
$$\chi_{iLn}^{E}(y_0) = 0. \tag{30}$$

Similarly, the right handed lepton $\chi_{iRn}^{E}(y)$ that lives on the visible brane is calculated

$$\chi_{iRn}^{E}(y) = N_{Rn}\, e^{\sigma/2}\left(J_{\frac{1}{2}+r}(e^{\sigma}\,x_{nR}) + c_R\,Y_{\frac{1}{2}+r}(e^{\sigma}\,x_{nR})\right), \tag{31}$$

by using the Dirac equation for KK mode leptons. Here c_R reads

$$c_R = -\frac{J_{r-\frac{1}{2}}(x_{nR})}{Y_{r-\frac{1}{2}}(x_{nR})}, \tag{32}$$

where N_{Rn} is the normalization constant and $x_{nR} = \frac{m_{Rn}}{k}$. Notice that the constant c_L, the nth KK mode mass m_{Ln} in eq.(28) and the constant c_R, the nth KK mode mass m_{Rn} in eq.(31) are obtained by using the boundary conditions eq.(27) and eq.(30), respectively. For $m_{L(R)n} \ll k$ and $kR \gg 1$ they are approximated as:

$$m_{Ln} \simeq k\pi\left(n - \frac{\frac{1}{2}-r}{2} + \frac{1}{4}\right)e^{-\pi k R},$$
$$m_{Rn} \simeq k\pi\left(n - \frac{\frac{1}{2}+r}{2} + \frac{1}{4}\right)e^{-\pi k R} \quad \text{for } r < 0.5,$$
$$m_{Rn} \simeq k\pi\left(n + \frac{\frac{1}{2}+r}{2} - \frac{3}{4}\right)e^{-\pi k R} \quad \text{for } r > 0.5. \tag{33}$$

References

[1] E. D. Commins et.al. *Phys. Rev. A* **50** (1994) 2960.

[2] J. Bailey et al, *Journ. Phys.* **G4** (1978) 345;

[3] Particle Data Group, D. E. Groom et.al. *European Phys. Journ.* **C15** (2000) 1.

[4] K. F. Smith et.al, *Phys. Lett.* **B234** (1990) 191, 2885; I. S. Altarev et. al, *Phys. Lett.* **B276** (1992) 242.

[5] I. B. Khriplovich, *Yad. Fiz.* **44** (1986) 1019 (*Sov. J. Nucl. Phys.* **44** (1986) 659); *Phys. Lett.* **B173** (1986) 193; A. Czarnecki and B. Krause, *Acta Phys. Polon.* **B28** (1997) 829; *Phys. Rev. Lett* **78** (1997) 4339.

[6] C. R. Schmidt and M. E. Peskin, *Phys. Rev. Lett.* **69** (1992) 410.

[7] E. Iltan, *Phys. Rev.* **D64** (2001) 013013.

[8] E. Iltan, *JHEP* **0404** (2004) 018.

[9] E. Iltan, *Eur. Phys.* **C44** (2005) 411.

[10] S, Weinberg, *Phys. Rev. Lett.* **58** (1976) 657; N. G. Deshpande and E. Ma, *Phys. Rev.* **D 16** (1977) 1583; for a recent review, see H. Y. Cheng, *Int. J. Mod. Phys.* **A 7** (1992) 1059; G. C. Branco and M. N. Rebelo, *Phys. Rev. Lett.* **B 160** (1985) 117; J. Liu and L. Wolfenstein, *Nucl. Phys.* **B 289** (1987) 1; C. H. Albright, J. Smith and S. H. H. Tye, *Phys. Rev. D.* **D 21** (1980) 711; Atwood. et. al., *Phys. Rev.* **D 51** (1995) 1034.

[11] E. Iltan, *J. Phys.* **G27** (2001) 1723.

[12] E. Iltan, *Phys.Rev.* **D65** (2002) 073013.

[13] E. Iltan, *JHEP* **065** (2003) 0305.

[14] B. Dutta, R. N. Mohapatra, *Phys. Rev.* **D68** (2003) 113008.

[15] V. Baryshevsky, hep-ph/0402209; O. Lebedev, K. A. Olive, M. Pospelov, A. Ritz, *Phys.Rev.* **C70** (2004) 016003; A. G. Rodriguez, M. A. H. Ruiz, A. Santiago, *Phys.Rev.* **D69** (2004) 073008; R. E. Mischke, AIP Conf.Proc.675:246-250,2003; M. Endo, M. Kakizaki, M. Yamaguchi, *Phys.Lett.* **B 583** (2004) 186; D. Kawall, F. Bay, S. Bickman, Y. Jiang, D. DeMille, *Phys. Rev. Lett.* **C92** (2004) 133007; I. Masina, C. Savoy, *Phys.Lett.* **B579** (2004) 99; J. Hisano, Y. Shimizu, *Phys.Lett.* **B581** (2004) 224-230; A. Syntfeld, H. Mach, W. Kurcewicz, B. Fogelberg, W. Pociennik, E. Ruchowska *Phys.Rev.* **C68** (2003) 024304; T. Suzuki, H. Sagawa, K. Hagino, *Phys.Rev.* **C68** (2003) 014317.

[16] A. G. Rodriguez, M. A. H. Ruiz, L. N. L. Noriega, *Mod. Phys. Lett* **A19** 2227 (2004); A. G. Rodriguez, M. A. H. Ruiz, B. J. Valdes, M. A. Perez , *Phys. Rev.* **D74** 053002 (2006); A. G. Rodriguez, M. A. H. Ruiz, M. A. Perez , *Int. J. Mod. Phys.* **A22** 3493 (2007).

[17] L. Randall, R.Sundrum, *Phys. Rev. Lett.* **83** 3370, (1999).

[18] L. Randall, R.Sundrum, *Phys. Rev. Lett.* **83** 4690 (1999);

[19] W. D. Goldberger, M. B. Wise, *Phys. Rev. Lett.* **D 83** 4922, (1999).

[20] S. Chang, J. Hisano, H. Nakano, N. Okada, M. Yamaguchi, *Phys. Rev.* **D62** 084025, (2000).

[21] A. Pomarol, *Phys. Lett.* **B486** 153, (2000).

[22] H. Davoudias, J. L. Hewett, T. G. Rizzo, *Phys. Lett.* **B473** 43, (2000).

[23] T. Gherghetta, A. Pomarol, *Nucl. Phys.* **B586** 141, (2000).

[24] B. Batell, T. Gherghetta, *Phys. Rev.* **D73**, 045016 (2006).

[25] S. J. Huber,C. A. Lee, Q. Shafi, *Phys. Lett.* **B531** 112, (2002).

[26] Y. Grossman, M. Neubert, *Phys. Lett.* **B474** 361, (2000).

[27] S. J. Huber, hep-ph/0211056, (2002).

[28] S. J. Huber, *Nucl. Phys.* **B666** 269, (2003).

[29] S. J. Huber, Q. Shafi, *Phys. Rev.* **D63** 045010, (2001).

[30] I. I. Kogan, S. Mouslopolous, A. Papazoglou, G. G. Ross, *Nucl. Phys.* **B615** 191, (2001).

[31] T. Gherghetta, A. Pomarol, *Nucl. Phys.* **B602** 3, (2001).

[32] K. Ghoroku, A. Nakamura, *Phys. Rev.* **D65** 084017, (2002).

[33] K. Agashe, G. Perez and A. Soni, *Phys. Rev.* **D71** 016002, (2005).

[34] K.Agashe, A. E. Blechman and F. Petriello, *Phys. Rev.* **D74** 053011, (2006).

[35] E. D. Pree, M. Sher., *Phys. Rev.* **D73**, 0955006 (2006).

[36] G. Moreau, J. I. S. Marcos, *JHEP.* **0603**, 090 (2006)

[37] F. Ledroit, G. Moreau, J. Morel, hep-ph/0703262 (2007).

[38] M. Deile, et. al., Muon g-2 Collaboaration, , *Nucl.Phys.Proc.Suppl.* **116** (2003) 215.

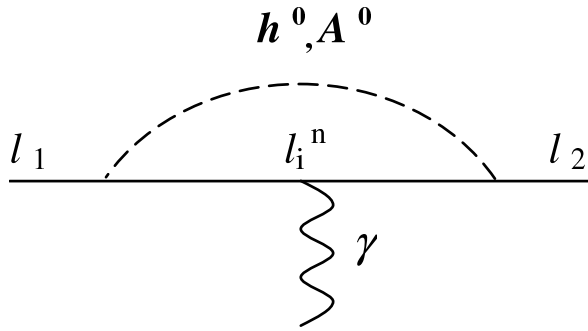[39] E. Iltan, H. Sundu, *Acta Phys.Slov.* **53** (2003) 17.

Figure 1: One loop diagrams contributing to EDM of l-lepton due to the neutral Higgs bosons h^0 and A^0 in the 2HDM. Wavy (dashed-solid) line represents the electromagnetic field (h^0 or A^0 fields-charged lepton fields and their KK modes).
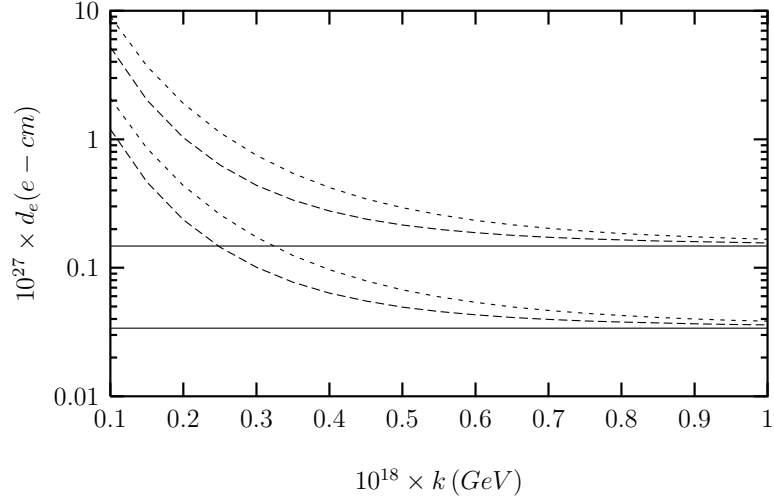
Figure 2: The parameter k dependence of the electron EDM d_e, for $\bar{\xi}^E_{N,\tau e} = 0.01\,(GeV)$. Here, the lower-upper solid (dashed, small dashed) line represents the d_e for $sin\,\theta = 0.1 - 0.5$ without KK modes (with KK modes set I, II).
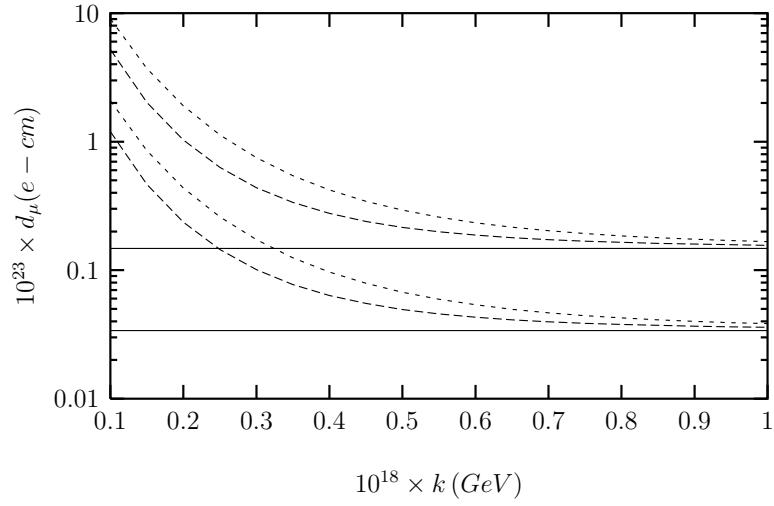

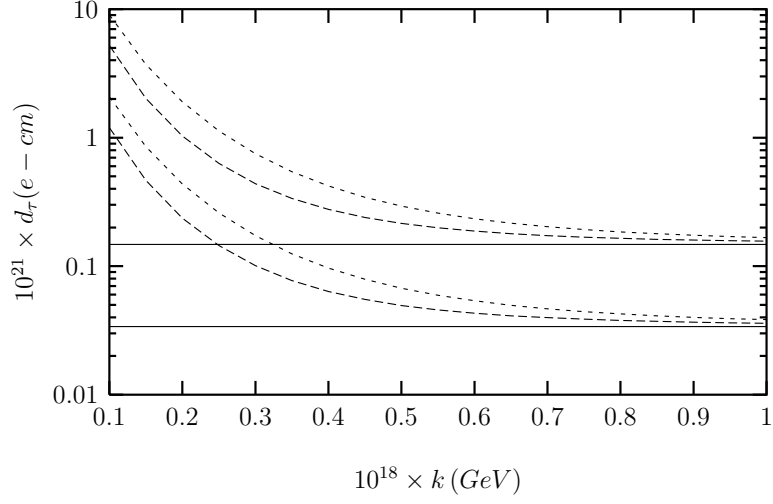Figure 3: The same as Fig. 2 but for d_μ and $\bar{\xi}^E_{N,\tau\mu} = 1\,(GeV)$.

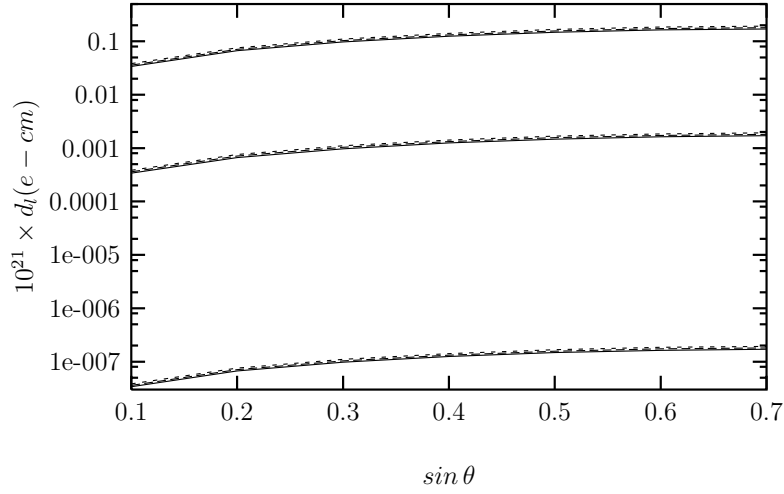Figure 4: The same as Fig. 2 but for d_τ and $\bar{\xi}^E_{N,\tau\tau} = 10\,(GeV)$.

Figure 5: The parameter $sin\,\theta$ dependence of the d_e; d_μ; d_τ, for $\bar{\xi}^E_{N,\tau e} = 0.01\,(GeV)$; $\bar{\xi}^E_{N,\tau\mu} = 1.0\,(GeV)$; $\bar{\xi}^E_{N,\tau\tau} = 10\,(GeV)$ and for $k = 10^{18}\,(GeV)$. Here, the lower-intermediate-upper solid (dashed, small dashed) line represents the d_e-d_μ-d_τ without KK modes (with KK modes set I, II).